Outline

Introduction

Block cipher model and security definition

Data Encryption Standard (DES)

Advanced Encryption Standard (AES)

Encryption modes of block ciphers

Authentication modes of block ciphers
Currently we are here...

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Encryption

Alice: sender, enciphers message to cryptogram using key
Bob: receiver, deciphers cryptogram to message using key
Eve: eavesdropper, does not have key
The one-time pad

message = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}

described as

keystream = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}

\oplus

cryptogram = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}
The one-time pad

message = 0 1 0 1 1 1 0 1
keystream = 1 1 1 1 1 0 0 0 0

\[ \oplus \]

cryptogram = 1 0 1 0 1 1 0 1
The one-time pad

message = 0 1 0 1 1 1 0 1
keystream = 1 1 1 1 0 0 0 0

\[ \oplus \]

cryptogram = 1 0 1 0 1 1 0 1

Provably secure if keystream is *fully random*
Stream cipher

Generates keystream bits $z_t$ from
- $K$: secret, typically 128 or 256 bits
- $IV$: initial value, for generating multiple keystreams per key
$z_t$ can be a bit or a sequences of bits, e.g. a 32-bit word
Example: DECT Stream Cipher

- In use in hundreds of millions of wireless phones
- 4 LFSRs with coprime lengths: large period
- top 3 clocked 2 or 3 times in between time steps $t$
Example: DECT Stream Cipher

- In use in hundreds of millions of wireless phones
- 4 LFSRs with coprime lengths: large period
- top 3 clocked 2 or 3 times in between time steps \( t \)
- practically broken with statistical key recovery attack
Example: RC4 [Ron Rivest] stream cipher

- State is array of 256 bytes
- Simple and elegant update function and output function
- Software-oriented

```plaintext
i := 0
j := 0
while GeneratingOutput:
i := (i + 1) mod 256
j := (j + S[i]) mod 256
swap values of S[i] and S[j]
K := S[(S[i] + S[j]) mod 256]
output K
endwhile
```

- Used in TLS and WEP
- Biases in keystream
- Practically broken in several use cases
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Permutation $B$ operating on $\mathbb{Z}_2^b$ with $b$ the block length
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Permutation $B$ operating on $\mathbb{Z}_2^b$ with $b$ the block length
- parameterized by a secret key: $B[K]$
- with an efficient inverse $B^{-1}[K]$
Block cipher definition

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- parameterized by a secret key: $B[K]$
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Computing $C = B[K](P)$ or $P = B^{-1}[K](C)$ should be
- efficient knowing the secret key $K$
- infeasible otherwise
Block cipher definition

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  - parameterized by a secret key: $B[K]$
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- Computing $C = B[K](P)$ or $P = B^{-1}[K](C)$ should be
  - efficient knowing the secret key $K$
  - infeasible otherwise
- Dimensions: block length $b$ and key length
Pseudorandom Permutation (PRP) security

Infeasibility to distinguish $B[K]$ from random permutation

Distinguishing should have expected effort that is out of reach
Pseudorandom Permutation (PRP) security

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Iterative block ciphers

- Data path (right): transforms
  - Iteration of a non-linear round function
  - ...that depends on a round key

- Key schedule (left)
  - Generates round keys from cipher key

Diagram:
- Key
- Data in
- KS Round → DP Round
- KS Round → DP Round
- KS Round → DP Round
- KS Round → DP Round
- KS Round → DP Round
- Data out
Iterative block ciphers

- Data path (right): transforms $P$ to $C$
  - iteration of a non-linear round function
  - ...that depends on a round key
Iterative block ciphers

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- Key schedule (left)
  - generates round keys from cipher key $K$
Substitution-permutation network (SPN)

- Round function in data path with two (or three) layers
  - Non-linear substitution layer: S-boxes applied in parallel
  - Permutation layer: moves bits to different S-box positions
  - Either key-dependent S-boxes or third layer of key addition
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Data encryption standard (DES)

- Standard by and for US government
- By National Institute for Standardization and Technology (NIST)
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  - complete block cipher specification
  - block length: 64 bits, key length: 56 bits
  - no design rationale
  - freely usable

Massively adopted by banks and industry worldwide
Dominated symmetric crypto for more than 20 years
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Data encryption standard: overview

Feistel data path

Linear key schedule

Data Encryption Standard (DES)
Data encryption standard: F-function

Variant of SPN with 4 layers:
Data encryption standard: F-function

Variant of SPN with 4 layers:

- expansion E: from 32 to 48 bits
- bitwise round key addition
- substitution: 8 different 6-to-4 bit non-linear S-boxes
- permutation P: moving nearby bits to remote positions

- clearly hardware-oriented
Non-ideal DES property: Weak Keys

- What happens if the cipher key is all-zero?
  - all round keys are all-zero
  - all rounds are the same
  - cipher and its inverse are the same
- Same is true for an all-one cipher key
- And two more keys due to symmetry in key schedule
- Weak key $K_w$:
  \[ \text{DES}[K_w] \circ \text{DES}[K_w] = I \]
- Also 6 semi-weak key pairs $(K_1, K_2)$
  \[ \text{DES}[K_1] \circ \text{DES}[K_2] = I \]
- Mostly of academic interest
Non-ideality in DES: Complementation Property

▶ What happens if we complement the cipher input?
- flip all bits in key
- flip all bits in plaintext

▶ In first round
- input to $F$ complemented so output of $E$ complemented
- round key also complemented so input to S-boxes unaffected
- output of $F$ unaffected

▶ Output of first round is simply complemented

▶ Repeat this until you reach the ciphertext

▶ Complementation property:

$$\text{DES}[K](P) = C \iff \text{DES}[\overline{K}](\overline{P}) = \overline{C}$$

▶ Reduces complexity of exhaustive key search from $2^{55}$ to $2^{54}$
Non-ideal DES properties: statistical attacks

- Two specific key-recovery attacks:
  - differential cryptanalysis: exploits difference propagation
  - linear cryptanalysis: exploits large \( P \)-to-\( C \) correlations

Differential cryptanalysis [Biham and Shamir, 1990]
- propagation of plaintext difference \( \Delta_p \) to ciphertext difference \( \Delta_c \)
- \( \text{DP}(\Delta_p, \Delta_a) \): probability that \( \Delta_p \) results in \( \Delta_c \)
- \( \exists \Delta_p, \Delta_c \) with \( \text{DP}(\Delta_p, \Delta_a) \) relatively high for all keys
- requires \(|Q_s| \approx 2^{47} \) (1000 TeraByte) chosen plaintexts

Linear cryptanalysis [Matsui, 1992]
- correlation between bits in plaintext \( u_p^T p \) and ciphertext \( u_c^T c \)
- \( \text{Corr}(u_p, u_a) \): correlation between \( u_p^T p \) and \( u_c^T c \)
- \( \exists u_p, u_c \) with \( \text{Corr}(u_p, u_c) \) relatively high for all keys
- requires about \(|Q_s| \approx 2^{43} \) (64 TeraByte) known plaintexts

Both break DES but still non-trivial to exploit in the field
The real problem of DES: the short key

Exhaustive key search: about $3.6 \times 10^{14}$ trials

More than 15 years ago: “software” cracking
- about 10,000 workstations
- 500,000 trials per second per workstation
- expected time: 7,200,000 seconds: 2.5 months

Applied in cracking RSA lab’s DES challenge, June 97

Cracking using dedicated hardware
- COPACOBANA RIVYERA (2008)
- costs about 10,000$
- board with 128 Spartan-3 5000 FPGAs.
- finds a DES key in less than a day

Short DES key is real-world concern!
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- Short DES key is real-world concern!
The solution: Triple DES (FIPS 46-2 and 46-3)

- Triple DES allows meet-in-the-middle attacks
- Three variants of Triple-DES
  - 3-key: 168-bit key, only option allowed by NIST
  - 2-key: 112-bit key by taking $K_3 = K_1$
    - still massively deployed by banks worldwide
  - 1-key: 56-bit key by taking $K_3 = K_2 = K_1$
    - falls back to single DES thanks to inverse in middle
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AES: the result of a competition

- January 1997: NIST announces the AES initiative
  - replacement of DES
  - open call for block cipher proposals
  - ...and for analysis, comparisons, etc.
- September 1997: official request for proposals
  - faster than Triple-DES
  - 128-bit blocks, 128-, 192- and 256-bit keys
  - specs, reference and optimized code, test vectors
  - design rationale and preliminary analysis
  - patent waiver
- Vincent Rijmen and I decided to submit a variant of Square
  - Most important change: multiple key and block lengths
  - We call it Rijndael
The AES competition

- First round: August 1998 to August 1999
  - 15 candidates at 1st AES conference in Ventura, California
  - analysis presented at 2nd AES conf. in Rome, March 1999
  - NIST narrowed down to 5 finalists using this analysis

- Second round: August 1999 to summer 2000
  - analysis presented at 3rd AES conf. in New York, April 2000
  - NIST selected winner using this analysis

- Criteria
  - security margin
  - efficiency in software and hardware
  - key agility
  - simplicity

- NIST motivated their choice in two reports
Rijndael design approach: the wide trail strategy

Round function with four layers, each with separate goal:
- nonlinear layer: S-boxes with high non-linearity
- dispersion layer: like $P$ in DES $F$-function
- mixing layer (absent in DES): linear local mixing
- round key addition

Mixing layer goals:
- each output bit depends on multiple input bits
- each small input difference propagates to multiple output bits

Quality of mixing layer quantified its branch number
- allows proving bounds related to resistance against LC/DC
- in combination with S-box layer and transposition layer
- link with theory of error-correcting codes
- optimum mix layer = maximum-distance-separable (MDS) code
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Rijndael

Block cipher with block and key lengths $\in \{128, 160, 192, 224, 256\}$
- set of 25 block ciphers
- AES limits block length to 128 and key length to multiples of 64
Rijndael

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  - all rounds are identical
  - ... except for the round keys
  - ... and omission of mixing layer in last round
  - parallel and symmetric

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Advanced Encryption Standard (AES)
Rijndael

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- Key schedule
  - Expansion of cipher key to round key sequence
  - Recursive procedure that can be done in-place
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- Manipulates bytes with simple operations in GF($2^8$)
The non-linear layer: SubBytes

Single S-box with two layers:

\[ y = x^{254} \text{ in } \mathbb{GF}(2^8) \]

- optimal non-linearity [Nyberg, Eurocrypt 1993]

Affine mapping: multiplication by 8×8 matrix in \( \mathbb{GF}(2) \)

- to have algebraic complexity, without it: \( xy = 1 \) for \( x \neq 0 \)
The non-linear layer: SubBytes

Single S-box with two layers:

\[ y = x^{254} \text{ in } GF(2^8) \]

- \( x^{#x} = 1 \) (Lagrange) so \( y = x^{-1} \text{ for } x \neq 0 \)
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Single S-box with two layers:
- \( y = x^{254} \) in GF\((2^8)\)
  - \( x \# x = 1 \) (Lagrange) so \( y = x^{-1} \) for \( x \neq 0 \)
  - optimal non-linearity [Nyberg, Eurocrypt 1993]
- Affine mapping: multiplication by \( 8 \times 8 \) matrix in GF\((2)\)
  - to have algebraic complexity, without it: \( xy = 1 \) for \( x \neq 0 \)
The mixing layer: MixColumns

- Same mapping applied to all 4 columns

\[
\begin{bmatrix}
2 & 3 & 1 & 1 \\
1 & 2 & 3 & 1 \\
1 & 1 & 2 & 3 \\
3 & 1 & 1 & 2
\end{bmatrix}
\]

- Multiplication by a 4 \(\times\) 4 circulant matrix in \(\text{GF}(2^8)\)

- Elements: 1, 1, \(x\) and \(x + 1\)

- Circulant MDS (\(B = 5\)) matrix with the simplest elements

- Inverse has more complex elements
The mixing layer: MixColumns

- Same mapping applied to all 4 columns
- Multiplication by a $4 \times 4$ circulant matrix in $\text{GF}(2^8)$
  - Elements: 1, 1, $x$ and $x + 1$
  - *circulant MDS ($\beta = 5$) matrix with the simplest elements*
  - Inverse has more complex elements
The dispersion layer: ShiftRows

- Each row is shifted by a different amount.
- Different shift offsets for higher block lengths.
- Together with MixColumns and SubBytes:
  - Full diffusion in two rounds.
  - $B_2 = 25$ active S-boxes in 4 rounds.
The dispersion layer: ShiftRows

- Each row is shifted by a different amount
- Different shift offsets for higher block lengths
- Together with MixColumns and SubBytes:
- 
  - full diffusion in two rounds
  - $B^2 = 25$ active S-boxes in 4 rounds
Round key addition: AddRoundKey

\[
\begin{array}{cccc}
a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\
a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\
a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\
a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \\
\end{array}
\] +
\[
\begin{array}{cccc}
k_{0,0} & k_{0,1} & k_{0,2} & k_{0,3} \\
k_{1,0} & k_{1,1} & k_{1,2} & k_{1,3} \\
k_{2,0} & k_{2,1} & k_{2,2} & k_{2,3} \\
k_{3,0} & k_{3,1} & k_{3,2} & k_{3,3} \\
\end{array}
\] =
\[
\begin{array}{cccc}
b_{0,0} & b_{0,1} & b_{0,2} & b_{0,3} \\
b_{1,0} & b_{1,1} & b_{1,2} & b_{1,3} \\
b_{2,0} & b_{2,1} & b_{2,2} & b_{2,3} \\
b_{3,0} & b_{3,1} & b_{3,2} & b_{3,3} \\
\end{array}
\]
Key schedule: 192-bit key, 128-bit block example

\[
k_0 \quad k_1 \quad k_2 \quad k_3 \quad k_4 \quad k_5 \quad k_6 \quad k_7 \quad k_8 \quad k_9 \quad k_{10} \quad k_{11} \quad k_{12} \quad k_{13} \quad k_{14} \quad k_{15} \quad \cdots
\]

Round key 0  Round key 1  Round key 2  \cdots

\[
k_{6n} = k_{6n-6} \oplus f(k_{6n-1})
\]

\[
k_i = k_{i-6} \oplus k_{i-1}, \quad i \neq 6n
\]

\(f\): AES S-box in parallel to 4 bytes followed by cyclic shift over 1 byte
# rounds: $6 + \max(\ell_k, \ell_b)$ with $\ell_k$ key and $\ell_b$ block length in 32-bit words

- last round has no MixColumns to make inverse similar to cipher
Rijndael symmetry

- Highly symmetric round function (as opposed DES)
  - SubBytes: 1 S-box instead of different ones
  - MixColumns: 1 MDS matrix with circulant symmetry
  - ShiftRows: bytes relative movement independent of position
  - round function minus key addition is shift-invariant
### Rijndael symmetry

- Highly symmetric round function (as opposed to DES)
  - **SubBytes**: 1 S-box instead of different ones
  - **MixColumns**: 1 MDS matrix with circulant symmetry
  - **ShiftRows**: bytes relative movement independent of position
  - Round function minus key addition is **shift-invariant**

- Very high symmetry in nonlinear part of S-box: \( y = x^{-1} \)
  - Representation of elements of \( \mathbb{GF}(2^8) \): choice of **basis**
  - Elements as degree < 2 polynomials with coeff. in \( \mathbb{GF}(2^4) \)
  - Can be done recursively
  - Called **tower fields**
Rijndael symmetry

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- Asymmetry:
  - Inverse is different and slightly more expensive
  - Key schedule has some symmetry, but much less
Rijndael implementation aspects

- Implementations can exploit symmetry

Software with table-lookups:
- 4 Kbytes of table
- 16 table-lookup + 16 XORs per round

Software in bitslice:
- rearrangement of the bits
- only bitwise Boolean instructions and shifts

Hardware:
- very suitable thanks to arithmetic in GF($2^n$) instead of ($\mathbb{Z}_{2^n}$, +)
- fully parallel: combinatorial logic with full round
- serial: logic for 1 S-box and 1 MixColumns matrix column
- S-box area/circuit depth trade-off by using tower fields
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Rijndael security status

- Cryptanalysis (in public domain)
  - all attacks, also on reduced-round, have huge data complexity
  - there is an (academic) attack against full-round AES:
    - biclique attacks [Bogdanov, Khovratovich, Rechberger, 2011]
    - $|Q_c| \approx 2^{126}$: factor 2 gain compared to exhaustive key search
    - gain evaporates when looking at complete picture
  - solid security status thanks to public scrutiny

- Implementation attacks: exploiting implementation weaknesses
  - timing attacks: cache misses in table-lookups
  - power analysis: exploiting dependence of current on data
  - electromagnetic analysis: same for EM emanations
  - fault attacks: exploiting forced faults

- Implementation attacks are the ones that matter in practice!
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Block cipher modes for encryption

- DES can encipher 8-byte messages, AES of 16-byte messages
  - what about longer and shorter messages?
  - two approaches: block encryption and stream encryption
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Block encryption modes
- split the message in blocks
- after padding last *incomplete* block if needed
- apply permutation $B[K]$ (keyed block cipher) to blocks in some way
Block cipher modes for encryption

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- Block encryption modes
  - split the message in blocks
  - after padding last incomplete block if needed
  - apply permutation $B[K]$ (keyed block cipher) to blocks in some way

- Stream encryption modes
  - build a stream cipher with a block cipher as building block
Block encryption modes

- Ideal: wide block encryption
  - each cryptogram bit depends on each message bit and vice versa
  - hard to build using a fixed-length block cipher
  - not online: cannot encipher long messages on the fly

- Electronic Code Book (ECB) mode
  - we consider only 16-byte messages
  - longer messages are split in 16-byte blocks
  - shorter messages padded to 16 bytes

- Cipher Block Chaining (CBC) mode
  - ECB randomized with what’s available
  - requires also split in 16-byte blocks and padding
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- Simplest padding: append zeroes
  - up to length multiple of block length (e.g. 16 bytes)
  - shortest possible padding
  - as such not usable for our purposes

- Decryption of cryptogram gives padded message

- Recovering message requires removing padding
  - send along message or padding length with cryptogram
  - impose padding is injective (or reversible)

- Simplest reversible padding: a single 1 and then zeroes
  - extends message in all cases
  - turns 16-byte message into 32-byte string

- Padding with exotic requirements
  - random-length padding: to hide message length
  - random padding: to add entropy

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Electronic CodeBook Mode (ECB)

Advantages
- simple
- parallelizable

Limitation: equal plaintext blocks → equal ciphertext blocks:
- likely to happen in low-entropy messages
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Cipher Block Chaining mode (CBC)

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- **ECB with plaintext block randomized by previous ciphertext block**
- First plaintext block randomized with **Initial Value (IV)**
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  - requires randomly generating and transferring **IV**
Cipher Block Chaining mode (cont’d)

Replacing IV randomness by $N$ nonce requirement: $IV = B[K](N)$
Cipher Block Chaining mode (cont’d)

Replacing IV randomness by $N$ nonce requirement: $IV = B[K](N)$

Properties of CBC
- encryption strictly serial, decryption can be parallel
- $IV$ must be managed and transferred
- security less than what one would think
Stream encryption: Output FeedBack mode (OFB)

- State $a_t$ consisting of two parts: fixed key $K$ and output $z_t$
  - Initialization: $z_{t-1} = IV$
  - State update: $z_t = B[K](z_{t-1})$
  - Output: just take $z_t$

Properties:
- strictly serial
- cycle lengths not known in advance
- no need for $B^{-1}$ (valid for all stream encryption)
Stream encryption: Output FeedBack mode (OFB)

Stream cipher with:
- State $a^t$ consisting of two parts: fixed key $K$ and output $z_t$
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- Stream cipher with:
  - State \( a \) consisting of two parts: fixed key \( K \) and counter \( c \)
  - Initialization: \( c_0 = IV \)
  - State update: \( c_t = c_{t-1} + 1 \), with \( c \) interpreted as an integer
  - Output: \( z_t = B[K](c_t) \)

- Properties:
  - Fully parallelizable
  - Cycle length \( 2^b \) with \( b \) the block length
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## Encryption modes: overview

<table>
<thead>
<tr>
<th></th>
<th>ECB</th>
<th>CBC</th>
<th>OFB</th>
<th>Counter</th>
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<tbody>
<tr>
<td>parallel encryption</td>
<td>y</td>
<td>n</td>
<td>n</td>
<td>y</td>
</tr>
<tr>
<td>parallel decryption</td>
<td>y</td>
<td>y</td>
<td>n</td>
<td>y</td>
</tr>
<tr>
<td>random access</td>
<td>y</td>
<td>y</td>
<td>n</td>
<td>y</td>
</tr>
<tr>
<td>requires $B^{-1}$</td>
<td>y</td>
<td>y</td>
<td>n</td>
<td>n</td>
</tr>
<tr>
<td>requires padding</td>
<td>y</td>
<td>y</td>
<td>n</td>
<td>n</td>
</tr>
<tr>
<td>full collapse if nonce violation</td>
<td>n</td>
<td>n</td>
<td>y</td>
<td>y</td>
</tr>
<tr>
<td>error propagation $C \rightarrow P$</td>
<td>y</td>
<td>y</td>
<td>n</td>
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</tbody>
</table>

**Legend:**
- random access: fast decryption of bits anywhere in the message
- error propagation: single-bit error in $C$ expands to $b$ bits in $P$
Currently we are here...

Introduction

Block cipher model and security definition

Data Encryption Standard (DES)

Advanced Encryption Standard (AES)

Encryption modes of block ciphers

Authentication modes of block ciphers
Message authentication code (MAC) functions

![Diagram of a MAC function]

- Input: key $K$ and arbitrary-length message $M$
- Output: $\ell$-bit MAC or tag $T$ with $\ell$ some length

**Applications:**
- Message authentication: append MAC to message
- Entity authentication: compute MAC over challenge

**Ideal behaviour:** pseudorandom function (PRF)
- Returns fully uncorrelated responses for different inputs

If ideal, $\Pr(\text{success})$ of forging a pair $M, T = \text{MAC}(K, M)$ is $2^{-\ell}$
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- Observation: in CBC encryption, $C_i$ depends on $m_1$ to $m_i$

- Idea:
  - Apply CBC encryption to (padded) message
  - take $T$ equal to last ciphertext block
  - throw away other blocks (essential for security)

- Broken for arbitrary-length messages
  - length-extension weakness
Cipher Block Chaining MAC mode (CBC-MAC)

Observation: in CBC encryption $C_i$ depends on $m_1$ to $m_i$

Diagram:
- $m_1$ and $k$ input to $E$, output is $m_2$
- $m_2$ and $k$ input to $E$, output is $m_3$
- $m_x$ and $k$ input to $E$, output is the result

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A fix of CBC-MAC: C-MAC

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- Addition of a constant before last application of $B[K]$.
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Authentication modes of block ciphers
Summary

- Block ciphers are keyed $b$-bit permutations
  - a different permutation $B[K]$ per key $K$ (and tweak $w$)
  - with an efficient inverse $B[K]^{-1}$
  - exhaustive keysearch should be best attack (complexity $2^{|K|-1}$)
- DES and AES are the most widespread block ciphers
  - constructed by iterating a simple round function
  - round has steps for non-linearity, mixing and transposition
- Block ciphers are versatile:
  - block encryption modes: e.g., ECB and CBC
  - stream encryption modes: e.g., OFB, counter and CFB
  - MAC computation modes: e.g., CBC-MAC and C-MAC
- Inverse permutation only used in block encryption modes