On the Practical Exploitability of Dual EC DRBG in TLS Implementations

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August 2, 2014

Random numbers are important

- Cryptography needs random numbers to generate long-term secret keys for encryption and signatures.
- Many schemes expect random (or pseudorandom) numbers, e.g.
  - ephemeral keys for DH key exchange,
  - nonces for digital signatures,
  - nonces in authenticated encryption.
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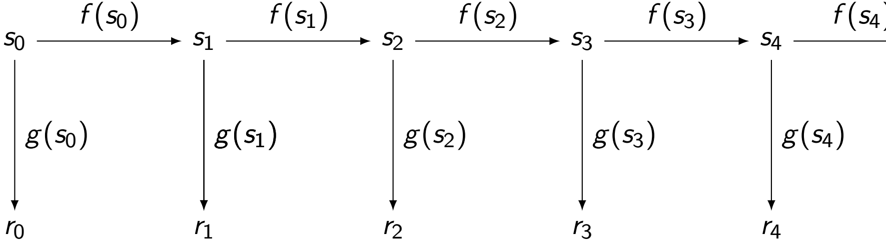
Snowden at SXSW:

> *[..] we know that these encryption algorithms we are using today work typically it is the random number generators that are attacked as opposed to the encryption algorithms themselves.*

Pseudo-random-number generators

Crypto libraries expand short seed into long stream of random bits.
Random bits are used as secret keys, DSA nonces, . . .

The usual structure, starting from short seed s_1:



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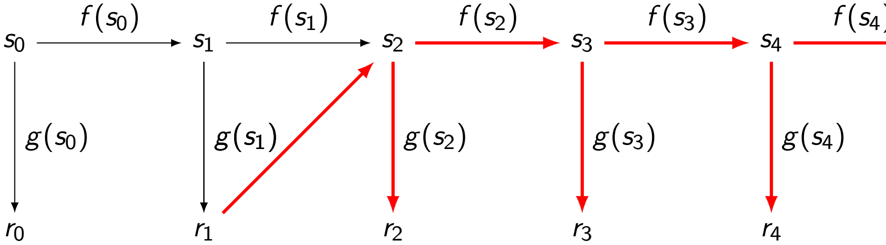

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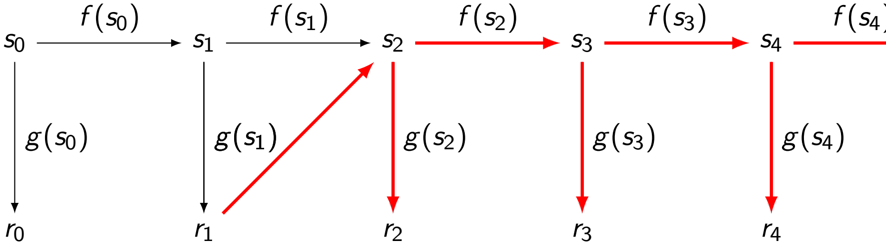

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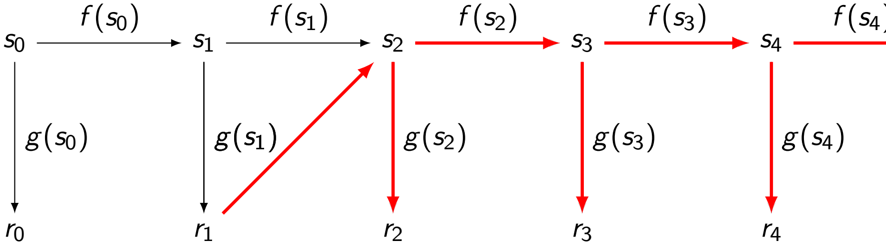

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3. Standardize this design of f, g.
4. Convince users to switch to this design: e.g., publish "security proof".

Elliptic curves

If P, Q are random points on a strong elliptic curve then it's hard to predict sP given sQ.

But if we know $P = kQ$ then it's easy: $sP = ksQ$.

Let's choose random Q, random k, define $P = kQ$.
Standardize this P; Q; $f(s) = sP$; $g(s) = sQ$.

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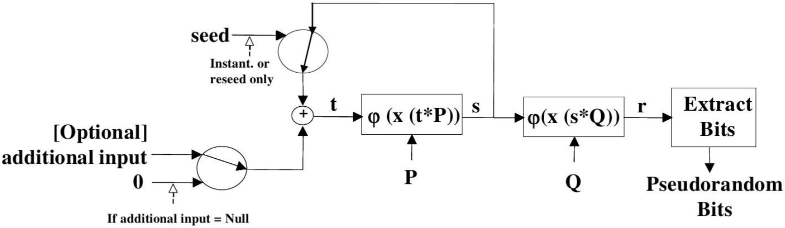
Solution: Let's throw away y and some bits of x.
Define $f(s) = x(sP)$, $g(s) = \phi(x(sQ))$ where ϕ omits 16 bits.
Not a big computation for us to recover sQ from $g(s)$.

DUAL_EC RNG: history part I

Earliest public source (?) June 2004, draft of ANSI X9.82:



Extract gives all but the top 16 bits ⇒ about 2^{15} points sQ match given string.

Claim:

Dual_EC_DRBG is based on the following hard problem, sometimes known as the "elliptic curve discrete logarithm problem" (ECDLP): given points P and Q on an elliptic curve of order n, find a such that $Q = aP$.

DUAL_EC RNG: common public history part II

Various public warning signals:

- Gjøsteen (March 2006): output sequence is biased.
  "While the practical impact of these results are modest, it is hard to see how these flaws would be acceptable in a pseudo-random bit generator based on symmetric cryptographic primitives. They should not be accepted in a generator based on number-theoretic assumptions."
- Brown (March 2006): security "proof"
  "This proof makes essential use of Q being random." If d with $dQ = P$ is known then $dR_i = S_{i+1}$, concludes that there might be distinguisher.
- Sidorenko & Schoenmakers (May 2006): output sequence is even more biased.
  Answer: Too late to change, already implemented.
- Shumow & Ferguson (August 2007): Backdoor if d is known.
- NIST SP800-90 gets appendix about choosing points verifiably at random,
  but requires use of standardized P, Q for FIPS-140 validation.

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NIST's DRBG Validation List: more than 70 validations of Dual_EC_DRBG;
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NIST re-opens discussions on SP800.90; recommmends against using Dual_EC.
RSA suggests changing default in BSAFE.

21 April 2014 NIST removes Dual EC from the standard.

How expensive is using the backdoor?

Rereading the standard:

" $x(A)$ is the x-coordinate of the point A on the curve, given in affine coordinates. An implementation may choose to represent points internally using other coordinate systems; for instance, when efficiency is a primary concern. In this case, a point shall be translated back to affine coordinates before $x()$ is applied."

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Given $r_i = \varphi(x(s_i Q))$, $r_{i+1} = \varphi(x(s_{i+1} Q))$, and NSA backdoor $d = log_P(Q)$.

1. Expand r_i to candidate $Q_i = s_i Q$, [50% chance; if fail move on to next candidate]
2. compute candidate $P_{i+1} = dQ_i$ and candidate $s_{i+1} = x(P_{i+1})$
3. check, $\varphi(x(s_{i+1} Q))$ against r_{i+1}. [if fail, goto 1.; else most likely done!]

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From the standard:
"For performance reasons, the value of outlen should be set to the maximum value as provided in Table 4."

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