

# Hash-based signatures V

Few-times signatures

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SAC – Post-quantum cryptography

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Reason for the large number in Goldreich/Levin: must never hit same leaf twice.

By the birthday paradox we need **2256** leaves,  
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$$H(m) = (h_0, h_1, \dots, h_{k-1}),$$

where each  $h_i \in \{0, 1, 2, \dots, t-1\}$  for some  $t$ .

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Let  $H(m_j) = (h_{j,0}, h_{j,1}, \dots, h_{j,k-1})$ .

$H$  is  $r$ -subset-resilient if given  $H(m_1), H(m_2), \dots, H(m_r)$

the probability of finding  $m'$  with  $H(m') = (h'_0, h'_1, \dots, h'_{k-1})$  and

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The same leaf public key can be used for  $r + 1$  signatures if  $H$  is  $r$ -subset-resilient.

# Few-times signature HORS

(Hash to Obtain Random Subset)

General parameters:

- ▶ Integer parameters  $k, t, \ell$ .
- ▶ Hash function  $H : \{0, 1\}^* \rightarrow \{0, 1\}^{k \cdot \log_2 t}$ .
- ▶ One-way function  $f : \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$ .

KeyGen:

- ▶ Picks  $t$  strings  $s_i \in \{0, 1\}^\ell$ , compute  $v_i = f(s_i)$  for  $0 \leq i < t$ .
- ▶ Public key  $P = (v_0, v_1, \dots, v_{t-1})$ ; secret key  $S = (s_0, s_1, \dots, s_{t-1})$ .

Sign  $m \in \{0, 1\}^*$ :

- ▶ Compute  $H(m) = (h_0, h_1, \dots, h_{k-1})$ , where each  $h_i \in \{0, 1, 2, \dots, t-1\}$ .
- ▶ Signature on  $m$  is  $\sigma = (s_{h_0}, s_{h_1}, s_{h_2}, \dots, s_{h_{k-1}})$ .

Verify:

- ▶ Compute  $H(m) = (h_0, h_1, \dots, h_{k-1})$  and  $(f(s_{h_0}), f(s_{h_1}), f(s_{h_2}), \dots, f(s_{h_{k-1}}))$ .
- ▶ Verify that  $f(s_{h_i}) = v_{h_i}$  for  $0 \leq i < k$ .