1. The integers modulo 30 form a monoid, \((\mathbb{Z}/30, \cdot, 1)\), with respect to multiplication.
   (a) Determine \((\mathbb{Z}/30)^\times\), i.e. the group of invertible elements.
   (b) Denote the class \(a + 30\mathbb{Z}\) by \(a\). Compute the cyclic submonoids \(\langle 3 \rangle\), \(\langle 6 \rangle\), and \(\langle 7 \rangle\) and determine for each of them the cycle length and the tail length.
2. Let \((G, \circ, e, x \mapsto \text{inv}(x))\) be a group and let \(X \subseteq G\).
   (a) Show that the normalizer of \(X\) in \(G\)
   \[ N(X, G) = \{ g \in G | g \circ X = X \circ g \} \]
   is a subgroup of \(G\).
   (b) Show that the centre of \(G\)
   \[ Z(G) = \{ g \in G | h \circ g = g \circ h \text{ for all } h \in G \} \]
   is a subgroup of \(G\).
3. Determine \(C(\{(1, 2, 3), (1, 3, 2)\}, S_3)\) and \(N(\{(1, 2, 3), (1, 3, 2)\}, S_3)\)
4. Let \(\circ\) be an operation on the rationals \(\mathbb{Q}\) defined as follows:
   \[ a \circ b = ab + 2(a + b) + 2, \]
   where addition and multiplication are the regular operations on \(\mathbb{Q}\).
   (a) Show that \(\mathbb{Q}\) is a monoid with respect to the operation \(\circ\). You may use that \((\mathbb{Q}, +, 0, x \mapsto -x)\) and \((\mathbb{Q}, \cdot, 1, x \mapsto x^{-1})\) are commutative groups.
   (b) Is \((\mathbb{Q}, \circ)\) commutative?
   (c) Determine the invertible elements.