1. The integers modulo 30 form a monoid, $\left(\mathbb{Z}/30, \cdot, 1\right)$, with respect to multiplication.
   (a) Determine $\left(\mathbb{Z}/30\right)^\times$, i.e. the group of invertible elements.
   (b) Denote the class $a + 30\mathbb{Z}$ by $a$. Compute the cyclic submonoids $\langle 3 \rangle$, $\langle 6 \rangle$, and $\langle 7 \rangle$ and determine for each of them the cycle length and the tail length.

2. Let $(G, \circ, e, x \mapsto \text{inv}(x))$ be a group and let $X \subseteq G$.
   (a) Show that the normalizer of $X$ in $G$
      $$N(X, G) = \{g \in G | g \circ X = X \circ g\}$$
      is a subgroup of $G$.
   (b) Show that the centre of $G$
      $$Z(G) = \{g \in G | h \circ g = g \circ h \text{ for all } h \in G\}$$
      is a subgroup of $G$.

3. Determine $C(\{(1, 2, 3), (1, 3, 2)\}, S_3)$ and $N(\{(1, 2, 3), (1, 3, 2)\}, S_3)$

4. Let $\circ$ be an operation on the rationals $\mathbb{Q}$ defined as follows:
   $$a \circ b = ab + 2(a + b) + 2,$$
   where addition and multiplication are the regular operations on $\mathbb{Q}$.
   (a) Show that $\mathbb{Q}$ is a monoid with respect to the operation $\circ$. You may use that
   $(\mathbb{Q}, +, 0, x \mapsto -x)$ and $(\mathbb{Q}, \cdot, 1, x \mapsto x^{-1})$ are commutative groups.
   (b) Is $(\mathbb{Q}, \circ)$ commutative?
   (c) Determine the invertible elements.